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THE HIGH TEMPERATURE DISPERSION EQUATION FOR LONGITUDINAL PLASMA OSCILLATIONS IN TAG ¹

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Abstract

The calculations in the temporal axial gauge (TAG) are revised and a new prescription is introduced to avoid the well-known TAG-singularity. With this prescription we use the TAG-formalism to calculate the one-loop dispersion equation for the longitudinal plasma oscillations in the high temperature limit and find the complete selfconsistency of TAG for pragmatic aims. Our result reproduces the earlier known dispersion equation obtained in covariant gauges and this equality explicitly demonstrates the gauge independence of the dispersion law in the high temperature limit and its reliability.

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1 Introduction

The recent results obtained for the Debye mass screening beyond the leading term [1-6] revived the interest to the selfconsistency of calculations made in an axial gauge, namely in the temporal axial gauge (TAG). Today (bearing in mind the need to have results beyond the perturbative expansions) it is a very actual problem since all the axial gauges due to the simple Slavnov-Taylor identities (like in QED) are singled out and very convenient when any nonperturbative calculations are performed. Using these gauges there is a chance to build explicitly the nonperturbative vertex function in accordance with the corrections made for the dressed gluon propagator. In doing in this manner one has an opportunity to keep explicitly the gauge covariance throughout all calculations and thus solving the selfconsistency problem of any nonperturbative approaches beyond the standard expansions. TAG where the gauge vector is parallel to the medium vector u_μ has the additional advantage in selecting the tensor structures which determine the Green function formalism and their amount is minimum in TAG. The latter is the main argument of doing TAG (between other axial gauge) to be more preferable for practice although it has one very essential defect: a pole at $p_4 = 0$. There were many attempts to eliminate this singularity [7-10] but all these attempts being or a rather complicated or very radical destroy all advantages found in TAG. But it is also well-known that the problem concerns (in any case on the one-loop level, but see also [11]) only the two sums which can be easily redefined to be correct [12,13]. Both these sums are well-known and have a rather simple form

$$T \sum_{p_4} 1/p_4 = 0, \quad T \sum_{p_4} 1/p_4^2 = 0 \quad (1)$$

where $p_4 = 2\pi nT$ with $n = 0, \pm 1, \pm 2$ and so on. Our goal is to show that TAG (after the new additional prescription is made) is a reliable gauge for any pragmatic aims and to repeat the important result (earlier found only in covariant gauges): the equation which defines the longitudinal plasma oscillations in the high temperature limit. In doing so we can check explicitly the gauge-independence of the high temperature dispersion equation for the plasma oscillations (there is no damping in this limit) and reliability of TAG.

2 The TAG formalism and the exact Π_{44} -function

As it was mentioned above the choice of the gauge vector n_μ to be parallel to the medium one u_μ considerably simplifies at the beginning all the Green function technique. The exact polarization tensor (for TAG) is determined by two scalar functions only [12]

$$\Pi_{\mu\nu}(k) = G \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) + (F - G) B_{\mu\nu} \quad (2)$$

and due to this fact the gluon propagator has a rather simple form

$$\mathcal{D}_{ij}(k) = \frac{1}{k^2 + G} \left(\delta_{ij} - \frac{k_i k_j}{\mathbf{k}^2} \right) + \frac{1}{k^2 + F} \frac{k_i^2 k_j}{k_4^2 \mathbf{k}^2} \quad (3)$$

The scalar functions $F(k)$ and $G(k)$ are defined as follows

$$G(k) = \frac{1}{2} \left(\sum_i \Pi_{ii} + \frac{k_4^2}{\mathbf{k}^2} \Pi_{44} \right), \quad F(k) = \frac{k^2}{\mathbf{k}^2} \Pi_{44} \quad (4)$$

and they should be calculated through the graph (or another) representation for Π . Due to a peculiarity of the temporal axial gauge the $\mathcal{D}_{44}(k)$ -function is completely eliminated from the formalism as well as the $\mathcal{D}_{4i}(k)$ and $\mathcal{D}_{i4}(k)$ -ones. The exact Slavnov-Taylor identity for the Γ_3 -vertex function has the same form as in QED

$$r_\mu \Gamma_{\mu\nu\gamma}^{abc}(r, p, q) = ig f^{abc} [\mathcal{D}_{\nu\gamma}^{-1}(p) - \mathcal{D}_{\nu\gamma}^{-1}(q)] \quad (5)$$

and namely this fact is doubtless an advantage of the axial gauge.

The exact graph representation for the gluon polarization tensor is well-known (see e.g. [12,13]) and contains (for any axial gauge) the standard four nonperturbative graphs. However there is one important simplification if one considers the Π_{44} -function only; in this case two "one-loop" nonperturbative graphs give at once its exact expression since the remaining graphs (the two very complicated ones) are equal to zero at the beginning (the unique situation which takes place only in TAG). Thus after some algebra being performed the exact expression for the Π_{44} -function is found to be

$$\begin{aligned} \Pi_{44}(q_4, |\mathbf{q}|) = & \frac{g^2 N}{\beta} \sum_{p_4} \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \mathcal{D}_{ii}(p) - \\ & - \frac{g^2 N}{2\beta} \sum_{p_4} \int \frac{d^3 \mathbf{p}}{(2\pi)^3} (2p + q)_4 [\mathcal{D}_{li}(p + q) \Gamma_{ij4}(p + q, -p, -q) \mathcal{D}_{jl}(p)] \end{aligned} \quad (6)$$

and we are going to calculate its leading order in the high temperature limit.

For calculating in the lowest order it is enough to put in Eq.(6) only the bare vertex which has a rather simple form

$$\Gamma_{4ji}^{abc}(q, r, p) = -igf^{abc}\delta_{ij}(r_4 - p_4) \quad (7)$$

and when all algebra being performed one can obtain the more simple expression for $\Pi_{44}(q_4, |\mathbf{q}|)$ which now is valid only on the one-loop level. This expression is found to be

$$\begin{aligned} \Pi_{44}(q_4, |\mathbf{q}|) &= \frac{g^2 N}{\beta} \sum_{p_4} \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \mathcal{D}_{ii}^{(0)}(p) - \\ &- \frac{g^2 N}{2\beta} \sum_{p_4} \int \frac{d^3 \mathbf{p}}{(2\pi)^3} (2p + q)_4^2 [\mathcal{D}_{li}^{(0)}(p + q) \mathcal{D}_{il}^{(0)}(p)] \end{aligned} \quad (8)$$

where $\mathcal{D}_{ij}^{(0)}(p)$ is the standard bare propagator for TAG

$$\mathcal{D}_{ij}^{(0)}(p) = \frac{1}{p^2} (\delta_{ij} + \frac{p_i p_j}{p_4^2}) \quad (9)$$

3 The high temperature limit of the longitudinal plasma oscillation in the leading order

Now a simple algebra should be performed within Eq.(8). Then we calculate a few sums and namely here we encounter the main TAG disadvantage: the well-known pole which should be redefined. After doing this we obtain a rather complicated expression which will be calculated in the so-called high-temperature limit. This is a special manner of calculation which reproduced only T^2 -terms but in a straightforward way.

Thus after all algebra being performed within Eq.(8) it can be put into the form

$$\begin{aligned} \Pi_{44}(q_4, |\mathbf{q}|) &= \frac{g^2 N}{\beta} \sum_{p_4} \int \frac{d^3 p}{(2\pi)^3} \left\{ \left(\frac{2}{p^2} + \frac{1}{p_4^2} \right) \right. \\ &- \frac{1}{2} (2p + q)_4^2 \left[\frac{1 + \cos^2(\mathbf{p} + \mathbf{q}|\mathbf{p})}{p^2(p + q)^2} + \frac{\cos^2(\mathbf{p} + \mathbf{q}|\mathbf{p})}{p_4^2(p + q)_4^2} \right] \end{aligned} \quad (10)$$

$$+ (1 - \cos^2(\mathbf{p} + \mathbf{q}|\mathbf{p})) \left(\frac{1}{p^2(p+q)_4^2} + \frac{1}{(p+q)^2 p_4^2} \right) \Big] \Big\}$$

where a new abbreviation is introduced to be

$$\cos^2(\mathbf{p} + \mathbf{q}|\mathbf{p}) = \frac{(\mathbf{p} + \mathbf{q}|\mathbf{p})^2}{(\mathbf{p} + \mathbf{q})^2 \mathbf{p}^2} \quad (11)$$

In what follows we calculate only the leading T^2 -term within Eq.(10). For this case (i.e. when the T^2 -term is only kept) the two last terms in Eq.(10) can be at once omitted since in the high temperature region due to the factor $(1 - \cos^2(\mathbf{p} + \mathbf{q}|\mathbf{p}))$ the T^2 -term is absent in their asymptotic behaviour. One can check this explicitly by taking into account the regularized value for the appropriate sum which is found to be

$$\frac{1}{\beta} \sum_{p_4} \frac{1}{p_4^2[(q+p)_4^2 + \omega^2]} = -\frac{(\omega^2 - q_4^2)n^{(B)}(\omega)}{\omega(\omega^2 + q_4^2)^2} \quad (12)$$

where $n^{(B)}(\omega) = [\exp(\beta\omega) - 1]^{-1}$ and then calculating the asymptotic behaviour for the integral over momenta in accordance with (10). To find Eq.(12) we use the new ansatz which is proposed here for treating any integral with a p_4 singularity. This ansatz is very close to the well-known Landshoff's α -prescription [7] but it is essentially modified due to an exponential factor. More precisely it has the form

$$\left[\frac{1}{p_4^2} \right] \longrightarrow \lim_{\alpha \rightarrow 0} \frac{\exp\left[-\frac{\alpha^2 T^2}{|p_4|^4}\right]}{p_4^2 + \alpha^2}, \quad \alpha > 0 \quad (13)$$

where we use (and it is very essential) the absolute value of p_4 in the exponential factor. Exploiting this new ansatz one can easily check Eq.(1) and we use it as well to calculate all the sums in Eq.(10). In particular we find that

$$\frac{1}{\beta} \sum_{p_4} \frac{(2p+q)_4^2}{p_4^2(p+q)_4^2} = 0 \quad (14)$$

and bearing in mind Eq.(14) the last term from the second line in Eq.(10) can be omitted as well. So, we arrive at the complicated (but well-defined)

sum which being treated as usual has the form

$$\begin{aligned}
I &= \frac{1}{\beta} \sum_{p_4} \int \frac{d^3 p}{(2\pi)^3} \frac{(2p + q)_4^2}{p^2(p + q)^2} \\
&= \int \frac{d^3 p}{(2\pi)^3} \frac{n^{(B)}(p)}{2|\mathbf{p}|} \left\{ \left[\frac{(2i|\mathbf{p}| + q_4)^2}{(i|\mathbf{p}| + q_4)^2 + |\mathbf{p} + \mathbf{q}|^2} + h.c. \right] \right. \\
&\quad \left. + \left[\frac{(2i|\mathbf{p}| + q_4)^2}{(i|\mathbf{p}| + q_4)^2 + |\mathbf{p} - \mathbf{q}|^2} + h.c. \right] \right\} \quad (15)
\end{aligned}$$

All other sums in Eq.(10) are trivial and now using Eq.(15) we can directly calculate the T^2 -term within Eq.(10) which reproduces the final result for this task. In doing so we introduce the dimensionless variable $z = |\mathbf{p}|/T$ (see the expression for $n^{(B)}(p)$ in Eq.(15)) and keep only the leading term which due to dimension of the integral in Eq.(15) is the T^2 -term. All other terms of this expansion reproduce the next-to-leading order terms and should be omitted. Using this ansatz the integral over $|\mathbf{p}|$ is calculated exactly and only the integral over the angular variables still remains to compare with the known expression. The final result has the form

$$\begin{aligned}
\Pi_{44}(q_4, |\mathbf{q}|) &= \frac{g^2 T^2 N}{6} \left\{ 1 - (3q_4^2 + \mathbf{q}^2) \right. \\
&\quad \times \left. \int_0^1 \frac{dx}{q_4^2 + \mathbf{q}^2 x^2} + 2q_4^2(\mathbf{q}^2 + q_4^2) \int_0^1 \frac{dx}{(q_4^2 + \mathbf{q}^2 x^2)^2} \right\} \quad (16)
\end{aligned}$$

and that is completely the same as previously known in the covariant gauges (see [14,15] and also [16]) Then doing as usual (with $\xi = \frac{\omega}{|\mathbf{p}|}$) one obtains that

$$\omega_{||}^2(\xi) = \xi^2 p_{||}^2(\xi), \quad 1 < \xi < \infty \quad (17)$$

where

$$p_{||}^2(\xi) = 3\omega_{pl}^2 \left\{ \frac{\xi}{2} \log \left[\frac{\xi + 1}{\xi - 1} \right] - 1 \right\} \quad (18)$$

and we can conclude that the entire (for any momenta) dispersion curve of the longitudinal plasma oscillations is gauge-independent, of course, if the leading T^2 -term is kept only (here $\omega_{pl}^2 = g^2 T^2 N/6$).

4 Conclusion

To summarize we have proposed here a new prescription which successfully treats all TAG-singularities and completely restores the reliability of the TAG-formalism for pragmatic aims. To prove this we demonstrate TAG-possibility by reproducing with a new prescription the high temperature dispersion equation for longitudinal plasma oscillations and find that these calculations are reliable and equal to the well-known results. Of course, from a theoretical point of view there is a number of unsolved questions (e.g. the periodicity problem) but in as much as TAG is not a rigorous gauge at the beginning (and despite on that we choose it) all other theoretical questions can be plainly forgotten. Thus gathering all well-known results one can conclude that there are no problems with applying TAG on the one-loop level and it is not clear today indeed such problems arise for multi-loop (nonperturbative) calculations where TAG has at once a number of the serious advantages. However it is also known that the next-to-leading order Debye mass found in TAG [3,4] has a power screening behaviour which is in contradiction with that obtained in covariant gauges ([1,2] and [5,6]). Of course, it is a serious problem, but there are no reasons to consider that the discrepancy found is a defect of TAG where all calculations are selfconsistent and do not suffer from any infrared divergencies. We should notice that this is not the case for calculations made in other gauges ([1,2] and [5,6]) which are very sensitive to the infrared cutoff which is usually associated with the so-called gluon magnetic mass. Nevertheless it is doubtful that the strong infrared sensitivity of the next-to-leading order Debye mass indeed should be taken place (although see [17]) and it is not excluded that namely this fact leads to a discrepancy with the calculations made in the different gauges.

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Appendix

Here we compare the calculations made with the different prescriptions (namely with the prescription proposed here and with another one (see Ref.[10]) for the sums of Eq.(1). Our calculation concerns the well-known sum

$$T \sum_{p_4} 1/p_4^2 = 0 \quad (19)$$

which should be equal zero to reproduce explicitly many well-known results found in other gauges. With the prescription proposed here this sum is found to be

$$\frac{1}{\beta} \sum_{p_4} \frac{1}{p_4^2} \longrightarrow \lim_{\alpha \rightarrow 0} \frac{1}{\beta} \sum_{p_4} \frac{\exp[-\frac{\alpha^2 T^2}{|p_4|^4}]}{p_4^2 + \alpha^2} = \lim_{\alpha \rightarrow 0} \frac{\exp[-\frac{T^2}{\alpha^2}]}{2\alpha} \text{cth}[\frac{\beta\alpha}{2}] = 0 \quad (20)$$

and indeed is equal to zero exactly (in any orders of T). However this is not the case if another prescription (as it was done in Ref.[10]) is used. In this case one finds

$$\begin{aligned} \frac{1}{\beta} \sum_{p_4} \frac{1}{p_4^2} &\longrightarrow \\ \lim_{\alpha \rightarrow 0} \frac{1}{\beta} \sum_{p_4} \frac{p_4^2}{(p_4^2 + \alpha^2)^2} &= \lim_{\alpha \rightarrow 0} \left\{ \frac{1}{4\alpha} (\text{cth}[\frac{\beta\alpha}{2}] - \frac{\beta\alpha}{2} \frac{1}{\sinh^2[\frac{\beta\alpha}{2}]}) \right\} = \frac{\beta}{12} \end{aligned} \quad (21)$$

which is the same as (20) only in the high temperature region . This means that the latter ansatz will generate additional (the next-to-leading order) terms in calculations made beyond the high temperature region. So these two prescriptions are not identical although in the simplest case (see e. g. [10]) this discrepancy (the additional next-to-leading order term) was eliminated by the dimensional regularization to obtain the well-known result for the Debye screening length. Of course, this may be not the case in a more complicated calculation where these additional terms can reproduce the nonvanishing expressions and give a different result.

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